Lecture 7 – 02/04/2025

Physics of optical cavities

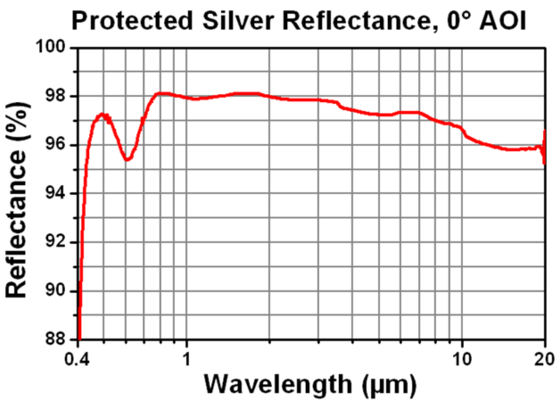
- Bragg mirrors, an alternative to metallic mirrors
- Fabry-Perot cavities: a reminder

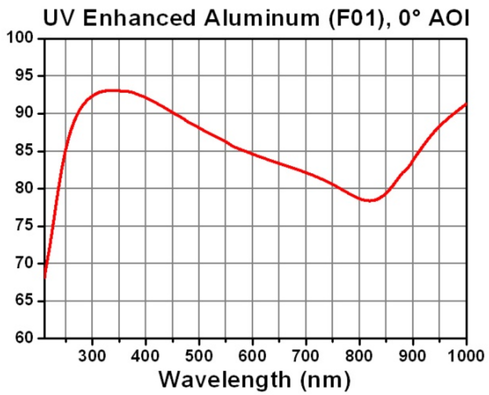
Light-matter interaction in microcavities

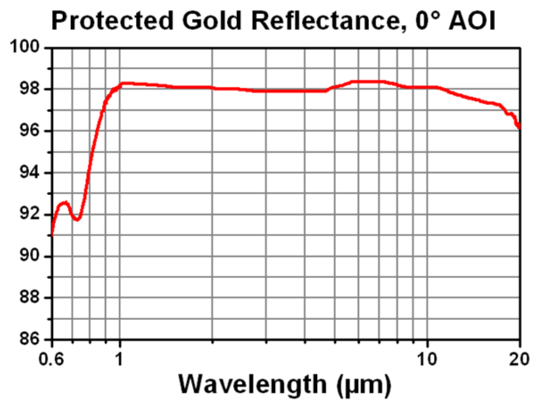
Physics of optical cavities

Limitations of metallic mirrors

Three metals are mostly used to realize mirrors in the visible and the IR: Ag, Al and Au





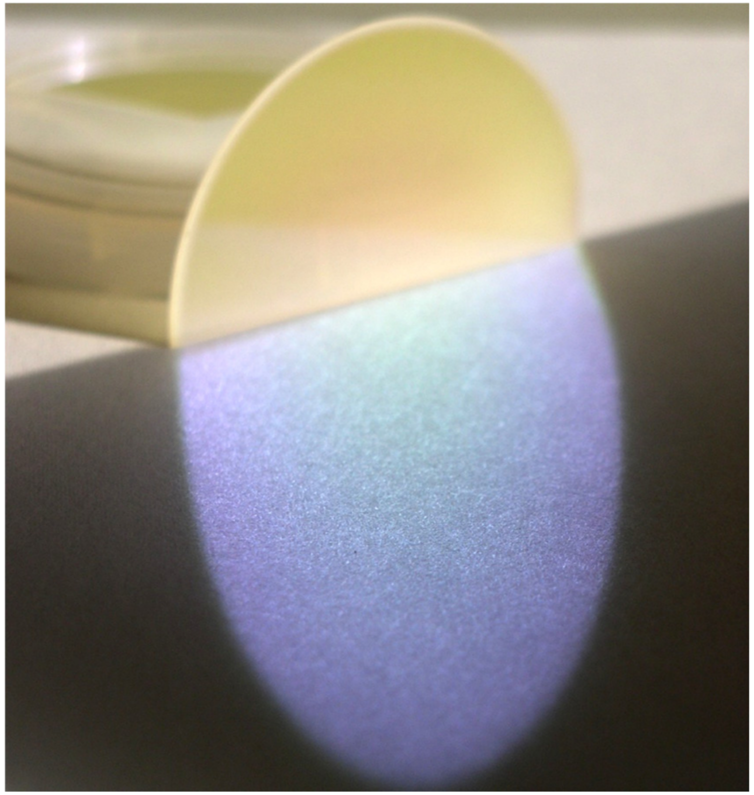


Issues: peak reflectivity (*R*) < 99% (major limitation in terms of quality factor), progressive decrease in *R* at short wavelengths due to proximity of plasmon frequency, structure (often amorphous) not compatible with the epitaxy of semiconductor layers

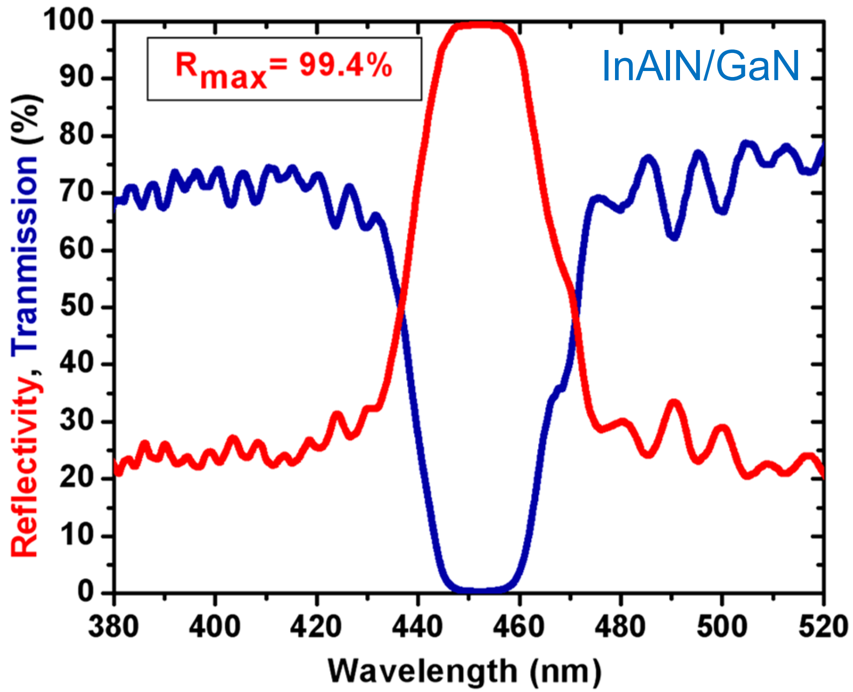
⇒ Which alternative?

Bragg mirrors or distributed Bragg reflectors (DBRs)

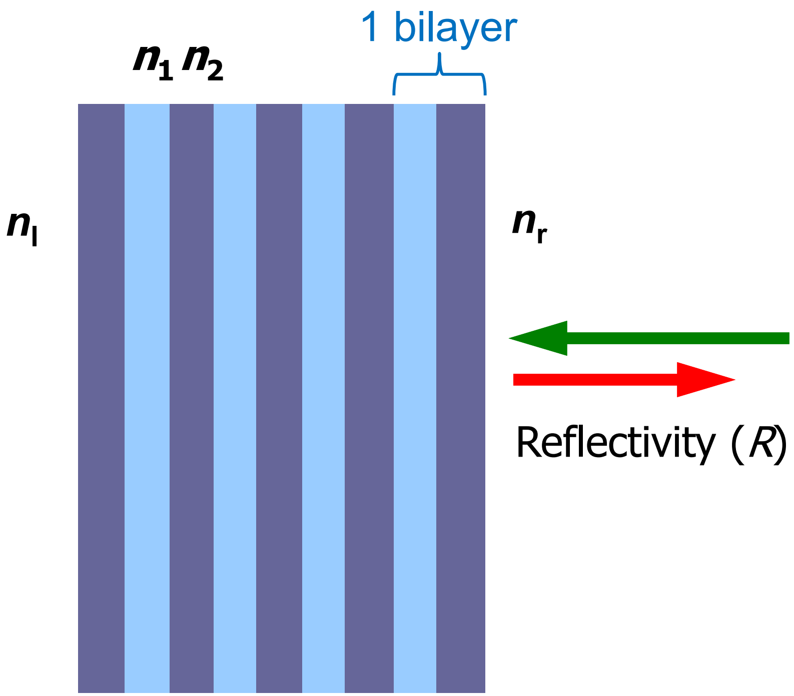
DBRs consist of a stack of quarterwave bilayers made from dielectric materials



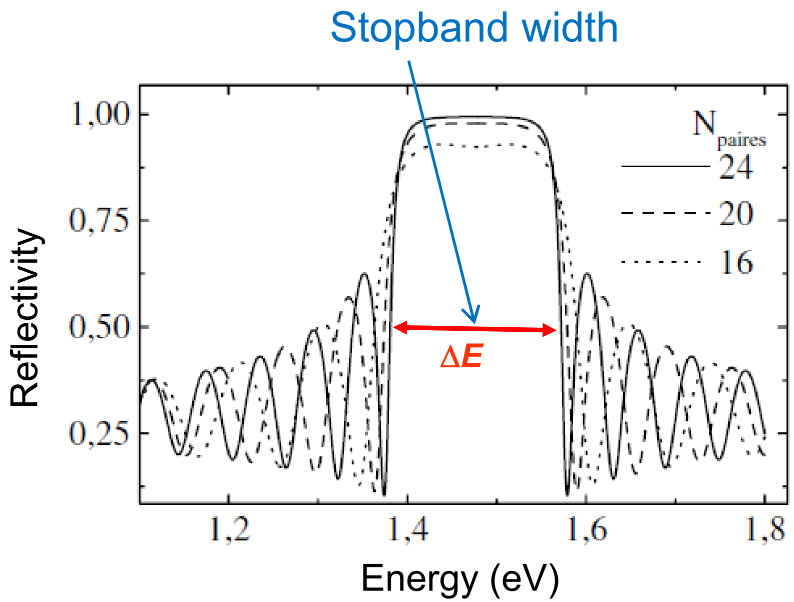
Alternative to metallic mirrors: higher peak reflectivity + compatibility with epitaxial requirements



Bragg mirrors

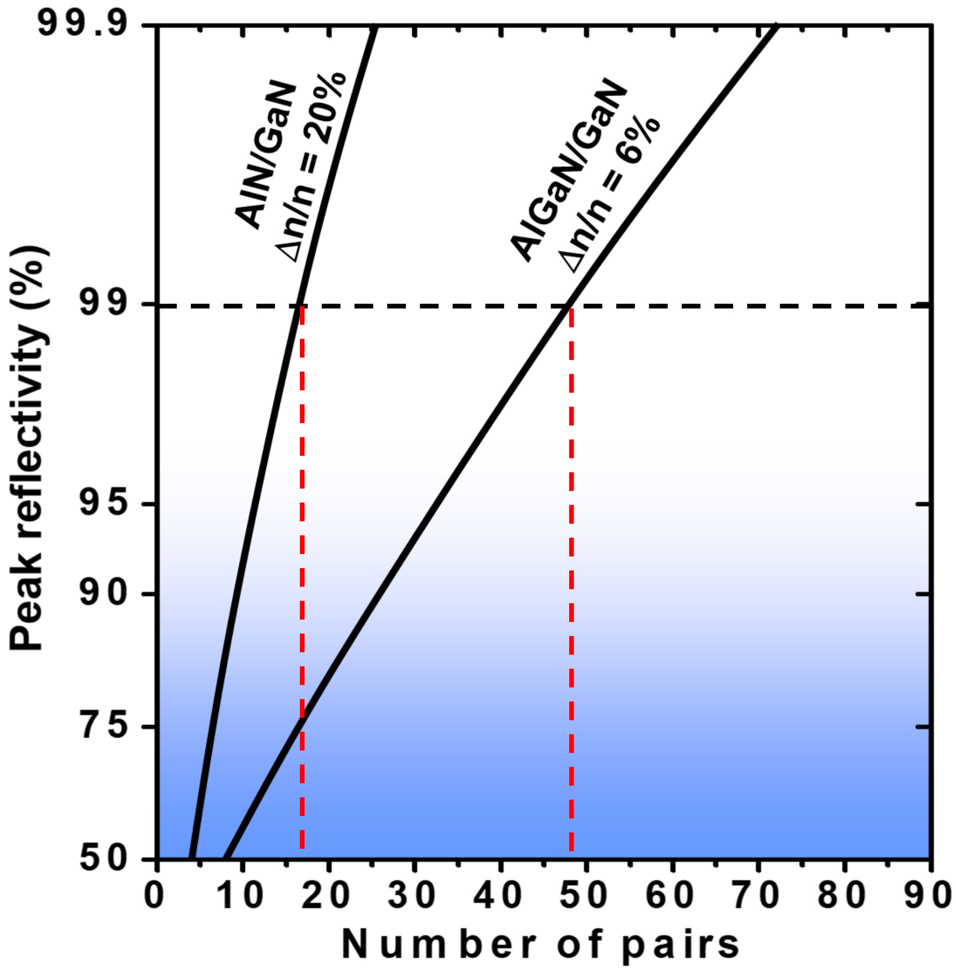


$\lambda/4 = nd$ (optical thickness)
$\rightarrow$ constructive interference

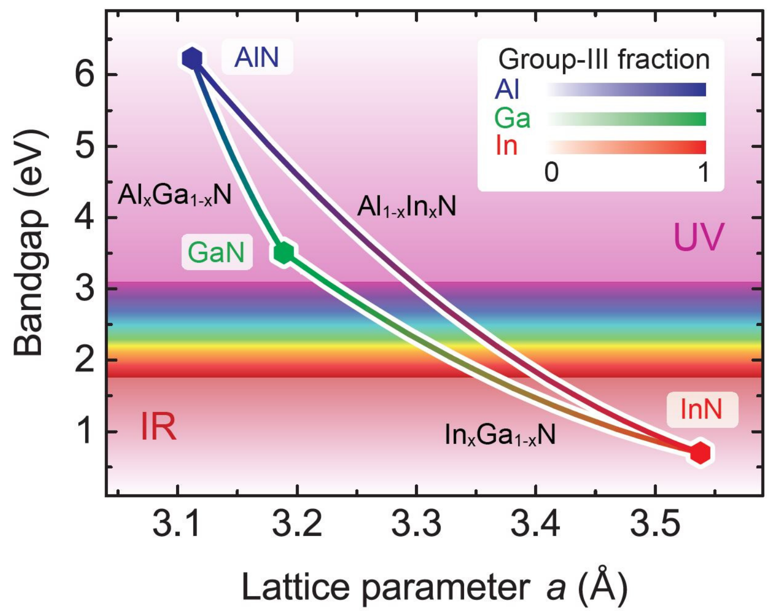


R and ΔE both depend on the refractive index contrast $(n_2\text{-}n_1)/n_1$

Bragg mirrors: examples



What could be a side effect when stacking many pairs/bilayers?



Strain buildup

⇒ source of macroscopic defects such as cracks (⇒ minimization of accumulated strain whenever possible)

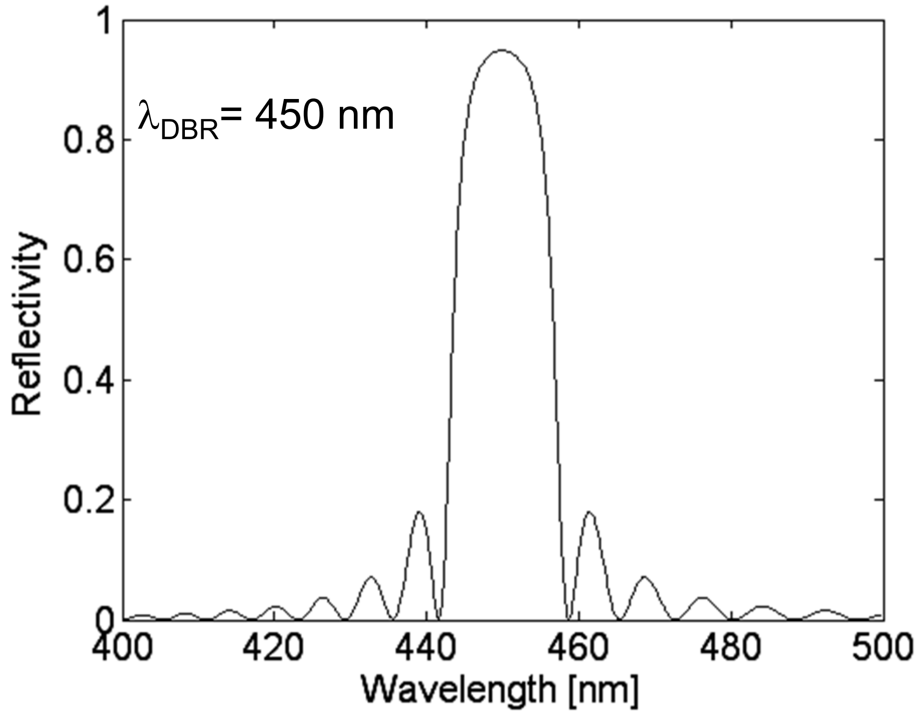
Bragg mirrors: examples

Keep in mind that the refractive index is a wavelength-dependent quantity!
(cf. Lecture 13, fall semester)

Reflectivity > 99%

DBR type	λ_{Bragg} [nm]	n_2	n_1	# of pairs
GaAs/AlAs	970	3.52	2.95	**11**
GaAs/AlO_x	970	3.52	1.6	**4**
GaN/$Al_{0.2}Ga_{0.8}N$	450	2.41	2.33	**64**
Si_3N_4/SiO_2	450	2	1.5	**9**

Bragg mirrors: examples



Stopband width

$$\frac{\Delta\lambda}{\lambda_0} = \frac{4}{\pi}\arcsin\left(\frac{\Delta n}{n_1 + n_2}\right)$$

Bragg mirrors: stopband width

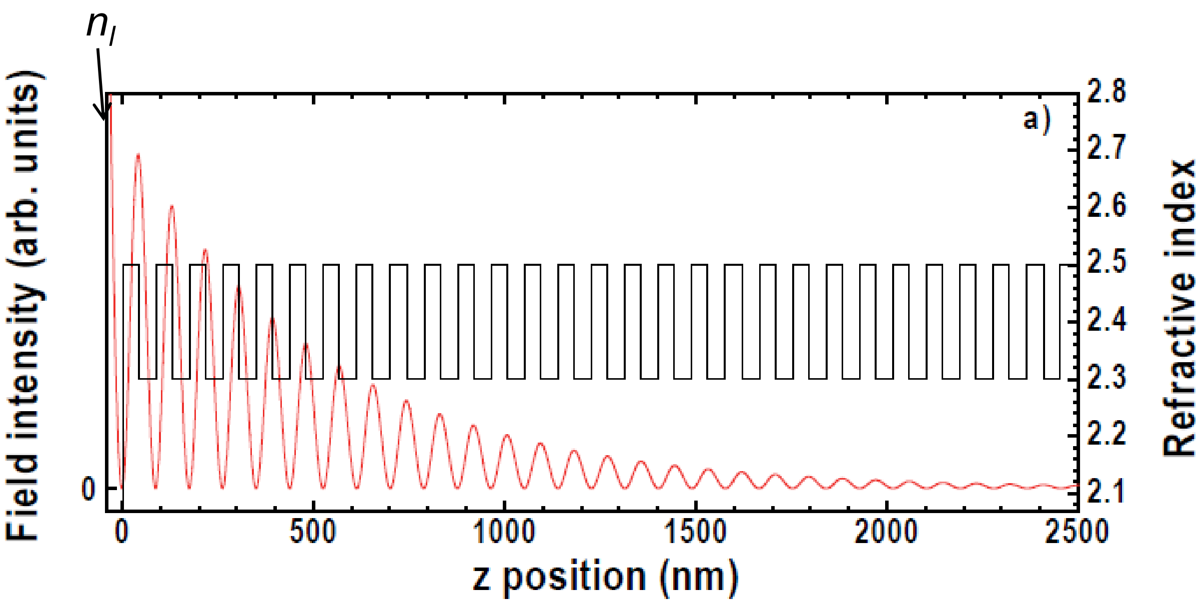
Stopband width

DBR type	λ_{Bragg} [nm]	n_2	n_1	$\Delta\lambda$ [nm]
GaAs/AlAs	**970**	3.52	2.95	**109**
GaAs/AlO_x	**970**	3.52	1.6	**475**
GaN/$Al_{0.2}Ga_{0.8}N$	**450**	2.41	2.33	**10**
Si_3N_4/SiO_2	**450**	2	1.5	**82**

$\Rightarrow$ Why such a stopband width value does not make any sense?

Bragg mirrors: penetration length

Penetration length $\equiv$ Equivalent location of an imaginary perfect mirror



$$L_{\mathrm{DBR}} = \frac{\lambda_0}{2} \frac{n_1 n_2}{n_l \left(n_1 - n_2 \right)}$$

Bragg mirrors: penetration length

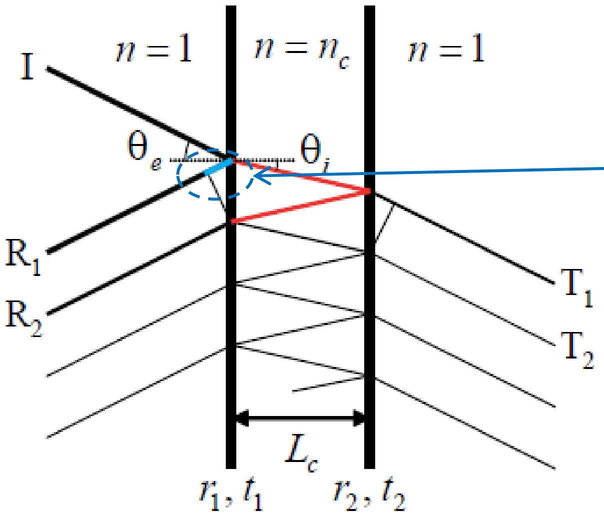
Penetration length

DBR type	λ_{Bragg} [nm]	n_2	n_1	# of pairs	L_{DBR} [nm]
GaAs/AlAs	**970**	3.52	2.95	**11**	**358**
GaAs/AlO_x	**970**	3.52	1.6	**4**	**117**
GaN/$Al_{0.2}Ga_{0.8}N$	**450**	2.41	2.33	**64**	**1218**
Si_3N_4/SiO_2	**450**	2	1.5	**9**	**199**

Fabry-Perot cavities

To be done during the series!

Transmitted electric field amplitude (E_t):



e: external
i: internal

$$E_t = E_i \frac{t_1 t_2}{1 - r_1 r_2 e^{i\delta}}$$

with δ the phase shift between transmitted and reflected waves:

$$\delta = \frac{4\pi n_c L_c \cos\theta_i}{\lambda}$$

The transmitted intensity is given by:

$$T = \left|\frac{E_t}{E_i}\right|^2 = \frac{(t_1 t_2)^2}{(1 - r_1 r_2)^2} \frac{1}{1 + \frac{4 r_1 r_2}{(1 - r_1 r_2)^2} \sin^2 \frac{\delta}{2}}$$

Zero loss case (*A* set to 0): *R* = 1-*T*

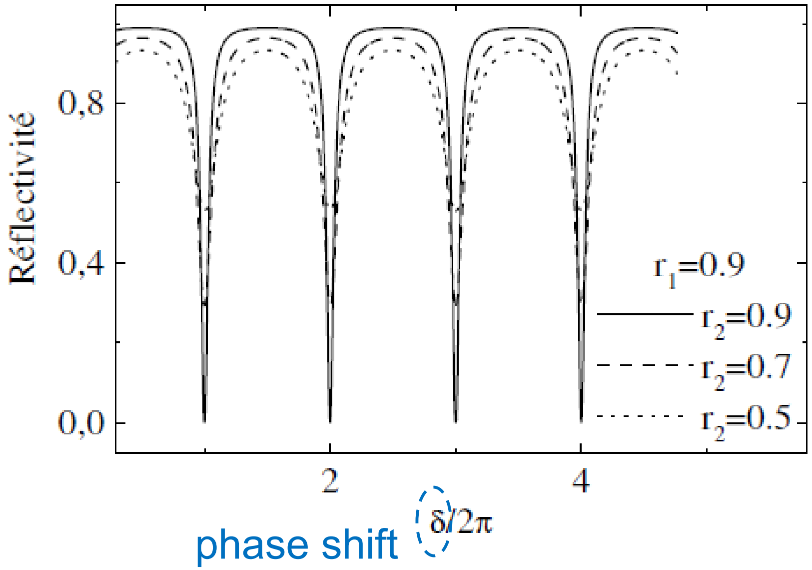
We also consider λ-independent r_i and t_i terms (+ n_c)

E_i: amplitude of incident electric field

Fabry-Perot cavities

Light confinement/trapping at specific wavelengths

Mode selectivity



NOTE: the higher the reflectivity of the mirrors the higher the mode selectivity (i.e., the finesse)

Cavity mode wavelength ($\delta = 2\pi q$ with q an integer), i.e., maximum of transmission:

$$2n_c L_c \cos\theta_i = q\lambda = \frac{\delta\lambda}{2\pi}$$

$$\begin{cases} \dfrac{I_r}{I_i} = \dfrac{f\sin^2(\delta/2)}{1+f\sin^2(\delta/2)} = R \\ \dfrac{I_t}{I_i} = \dfrac{1}{1+f\sin^2(\delta/2)} = T \end{cases}$$

Valid in the limit where $t_1 t_2 = 1 - r_1 r_2$ (i.e., in the zero-loss case), which is inherited from Stokes relations[1]

where $$f = \frac{4r_1r_2}{(1-r_1r_2)^2}$$

[1]See, e.g., *Optics*, 3rd edition, by E. Hecht (Addison Wesley, Reading, 1998) or *Principles of Optics*, 7th edition, by M. Born and E. Wolf (Cambridge University Press, Cambridge, 1999)

Figures of merit of optical cavities

Under normal incidence, the **cavity mode linewidth** is given by:

$$\gamma_c = \frac{c}{n_c L_c} \frac{1 - r_1 r_2}{\sqrt{r_1 r_2}}$$

Cavity finesse (spectral selectivity):

$$F = \frac{\pi \sqrt{r_1 r_2}}{1 - r_1 r_2}$$

Ratio between the spectral separation between consecutive modes and the cavity mode linewidth

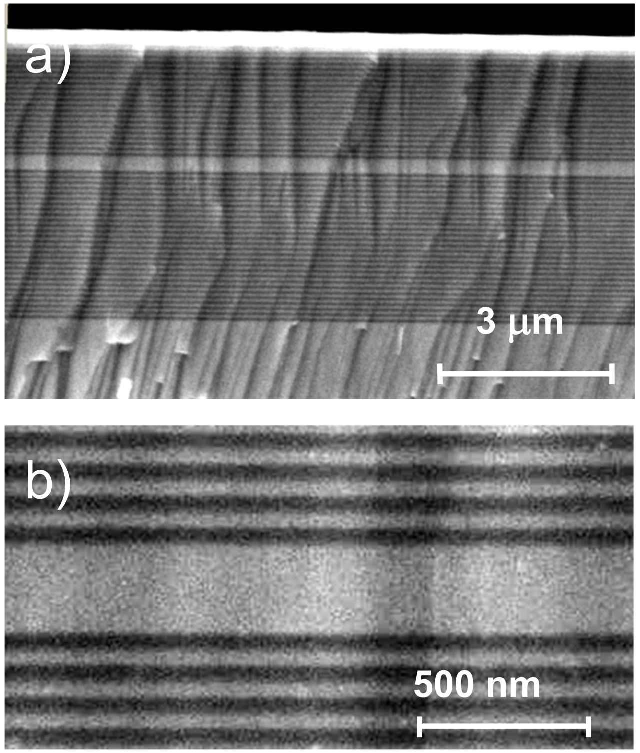
Quality factor:

$$Q = \frac{\lambda}{\Delta\lambda} = \omega \tau_c$$

τ_c is the cavity photon lifetime, i.e., the storage time of a photon in the cavity before it leaks out!

Planar microcavities

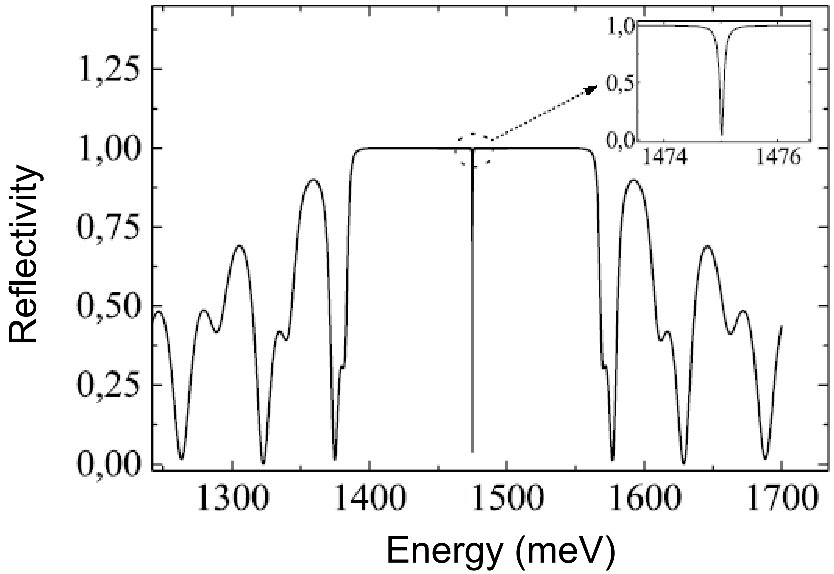
Cavity with a small thickness (on the order of the wavelength) **made from dielectrics**



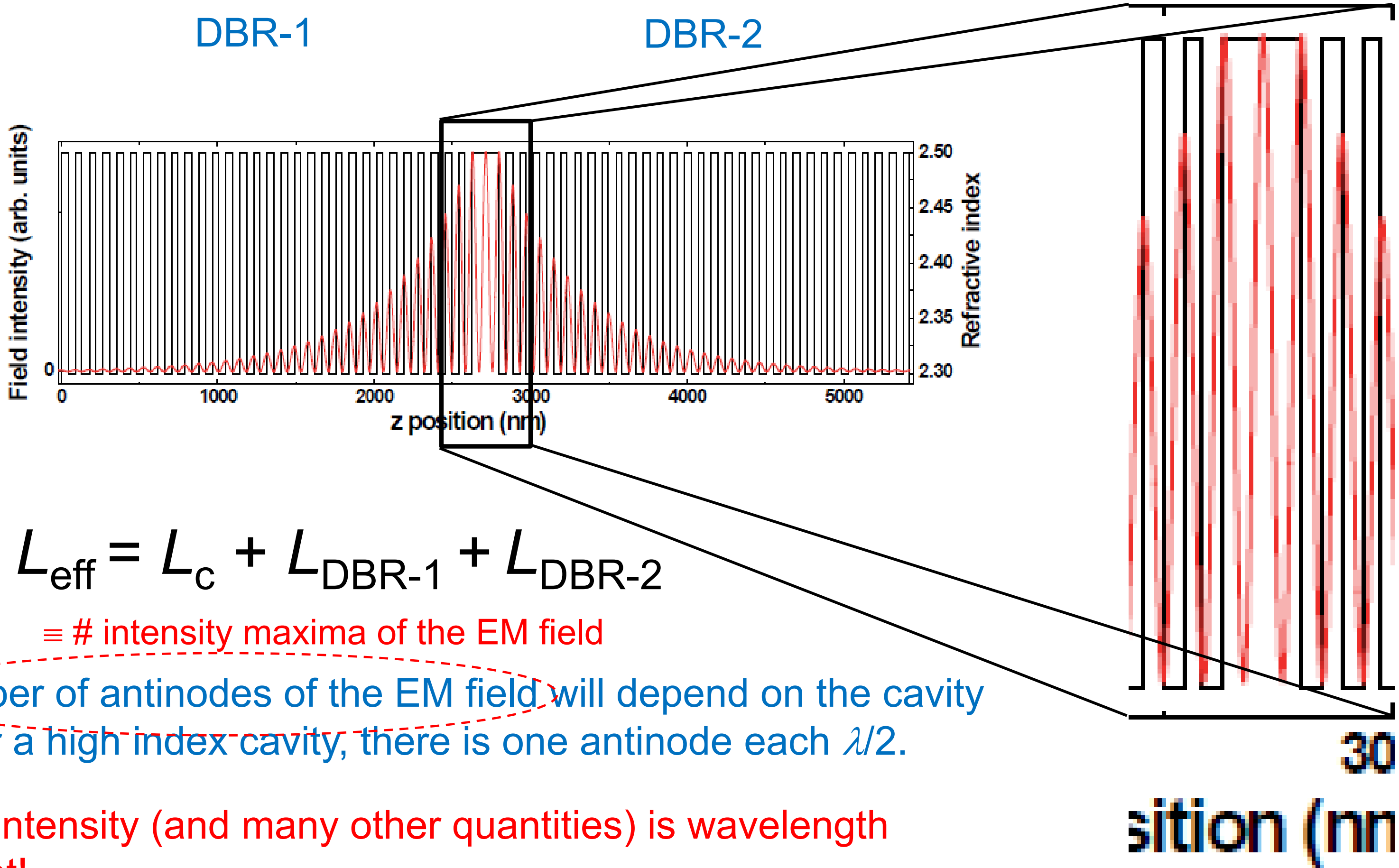
Since the mode wavelength is given by:

$$2n_c L_c \cos\theta_i = q\lambda$$

the cavity thickness under normal incidence is equal to $q\lambda/2$



Planar microcavities



$$L_{\text{eff}} = L_c + L_{\text{DBR-1}} + L_{\text{DBR-2}}$$

≡ # intensity maxima of the EM field

The number of antinodes of the EM field will depend on the cavity order. For a high index cavity, there is one antinode each $\lambda/2$.

The field intensity (and many other quantities) is wavelength dependent!

Planar microcavities: dispersion curve

To get the mode dispersion, we need to express the **total wavevector inside the cavity** (k_c) as a function of its normal ($k_\perp$) and parallel ($k_\parallel$) components.

The interference condition applied to the wavevector in the normal direction leads to:

$k_\perp 2L_c = 2q\pi$ where we use the fact that under normal incidence: $k_0 = k_\perp n_c$ and $k_0 = \dfrac{2\pi}{\lambda}$

Values in vacuum

The total wavevector of the resonant wave inside the cavity is thus:

$$k_{c,\,q} = \sqrt{{k_\parallel}^2 + {k_{c\perp}}^2} = \sqrt{{k_\parallel}^2 + \left(\frac{q\pi}{L_c}\right)^2}$$

$$E_q = \frac{\hbar c}{n_c}\frac{q\pi}{L_c}\sqrt{1 + \left(\frac{L_c k_\parallel}{q\pi}\right)^2}$$

Usually $k_{//} << \dfrac{q\pi}{L_c}$ ⇒

$$E_q = \frac{q\pi\hbar c}{n_c L_c}\left(1 + \frac{1}{2}\left(\frac{L_c k_\parallel}{q\pi}\right)^2\right) = E_{q,0} + \frac{\hbar^2 {k_\parallel}^2}{2m^*_{ph}}$$

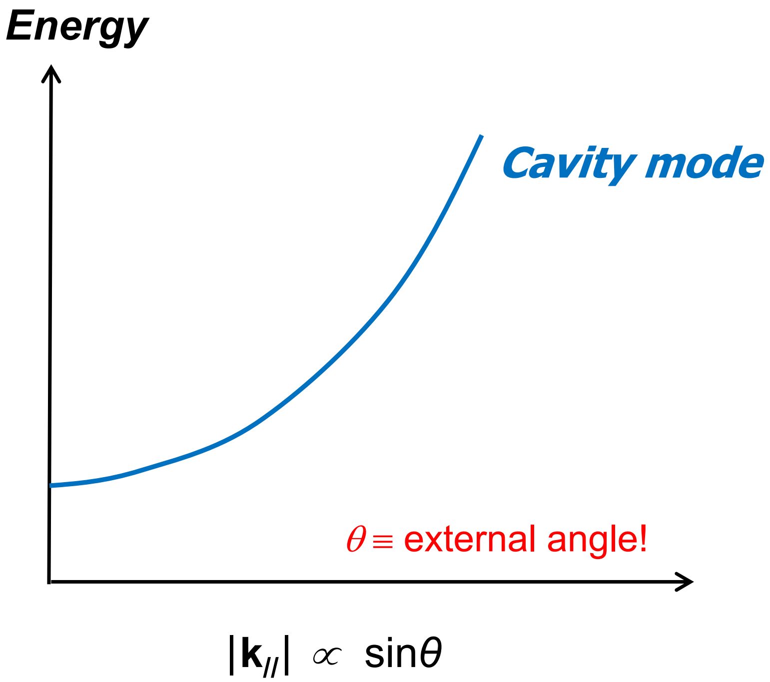
Parabolic dispersion

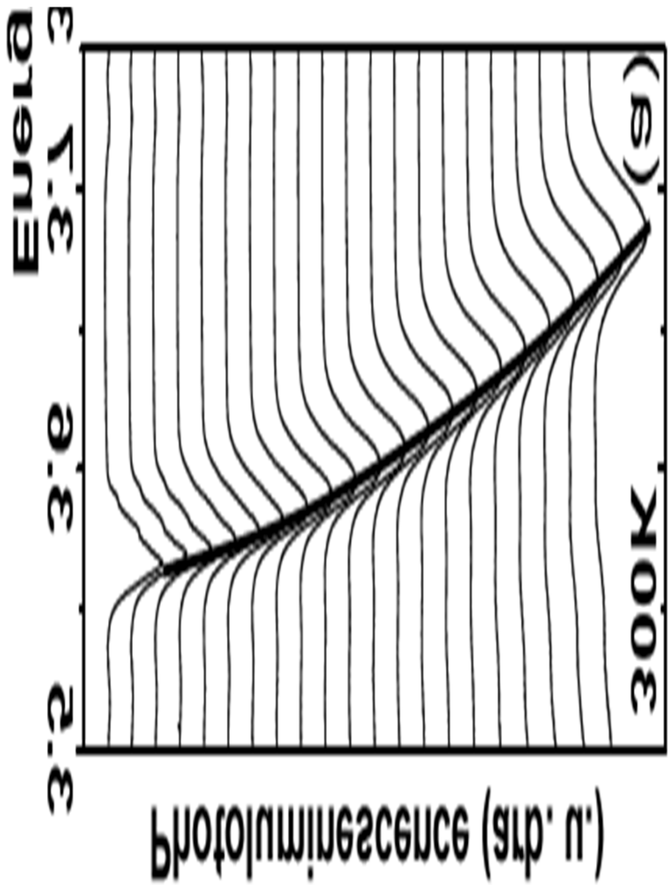
Effective mass of the cavity photon

$$m^*_{ph} = \frac{\hbar n_c q\pi}{cL_c}$$

Planar microcavities: dispersion curve

Dispersion curve





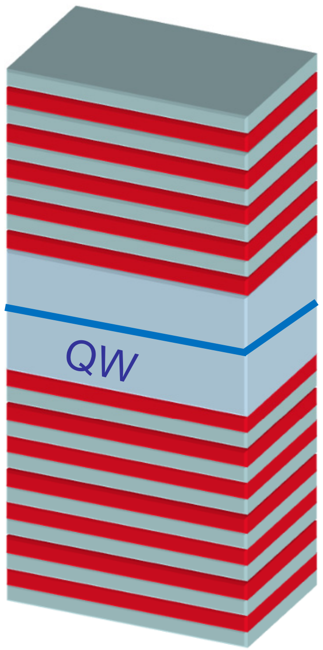
Very small cavity photon effective mass

$$m^{*}_{\text{ph}} \sim 10^{-5} m_0$$

Light-matter interaction in microcavities

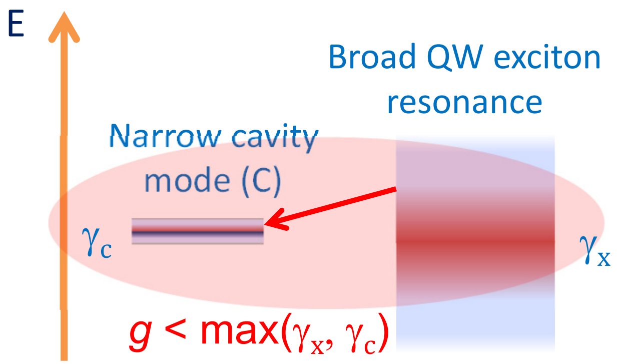
Microcavities: light-matter interaction

Semiconductor microcavity



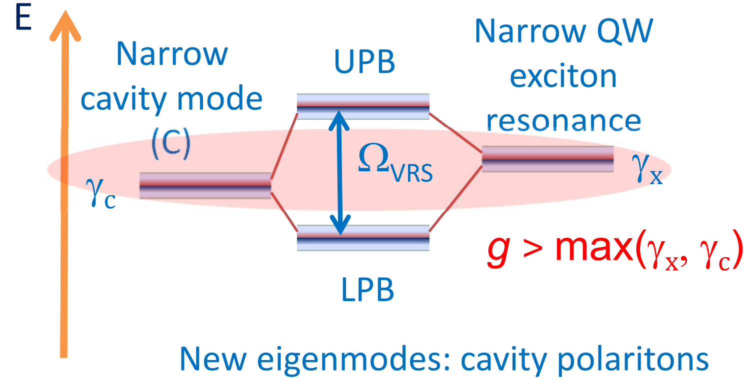
Cavity (MC) + oscillator (QW excitons)

Light-matter interaction



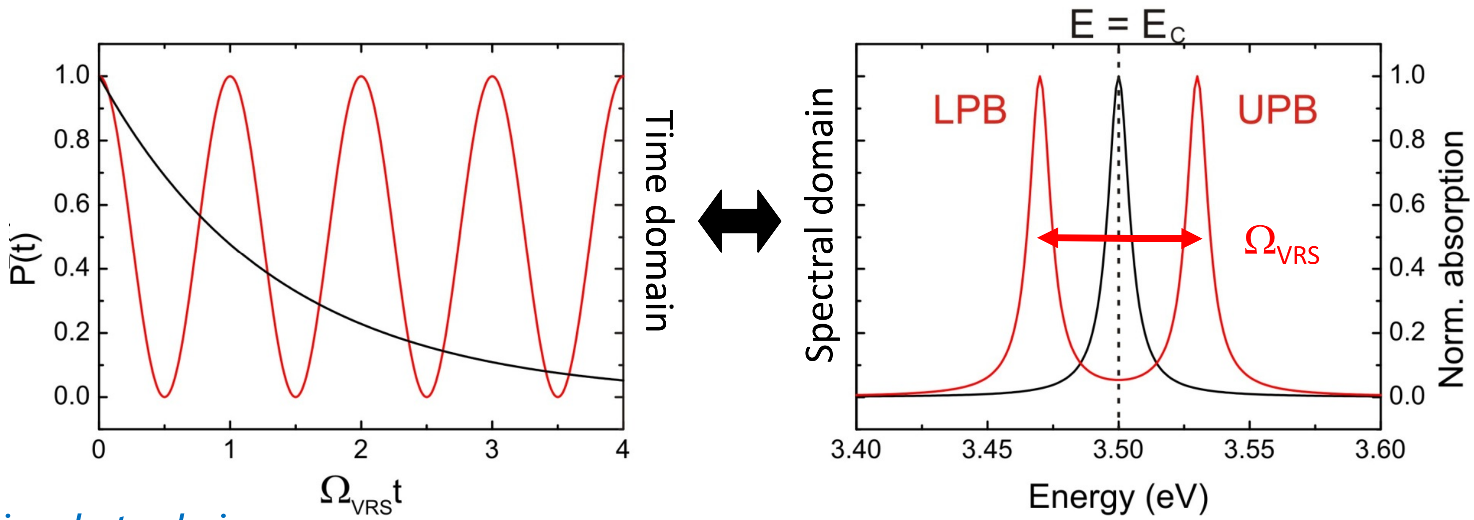
Perturbative (Fermi's golden rule)

Weak coupling regime (WCR)



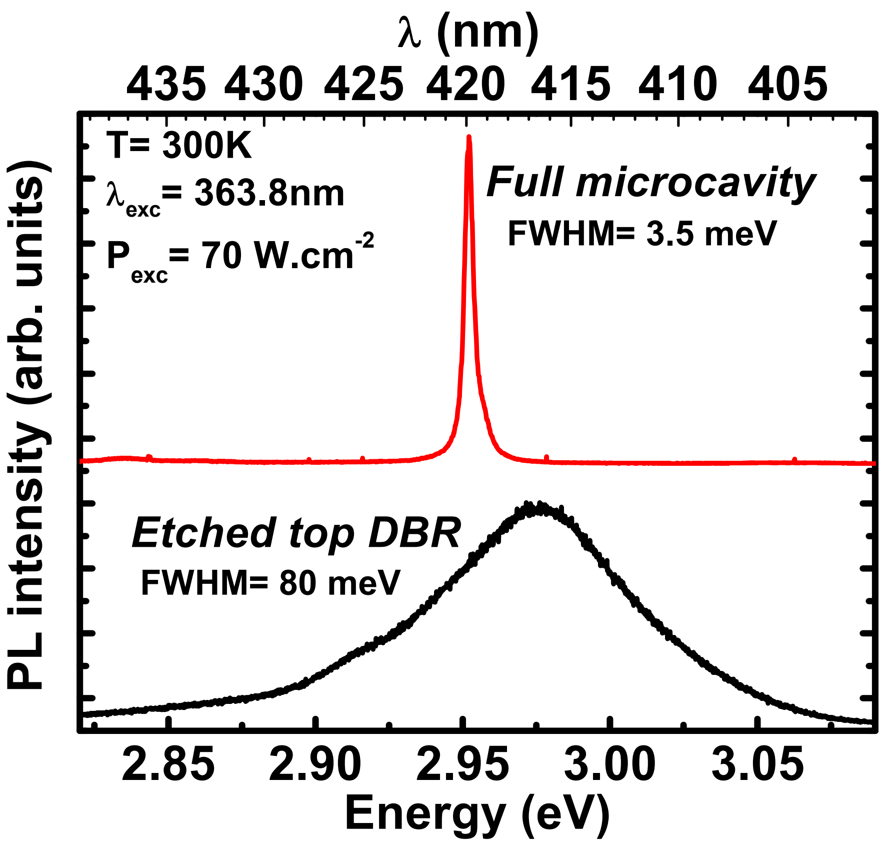
Non perturbative (correlated states)

Strong coupling regime (SCR)



Weak coupling regime

Spectral selectivity



Cavity photon storage at specific wavelengths ⇒ the cavity acts as a spectral filter

Case of a QW with a broad luminescence linewidth in a cavity